Semiclassics and Topological Aspects of the Quantum-Classical Transition

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The process by which a classical dynamical system emerges as a sufficient approximation to a quantum dynamical system has been a major topic of discussion since the inception of quantum mechanics. The singular nature of the semiclassical limit lies at the center of this debate. Classical behavior cannot emerge as the smooth limit of a closed quantum system with a nonlinear Hamiltonian as these classical evolutions violate the unitary symmetry of quantum mechanics. Moreover, the symplectic geometry of a chaotic classical phase space generates infinitely fine structures which the uncertainty principle prevents a quantum dynamical system from tracking at late times. Thus the pathologies associated with the semiclassical limit are most dramatic in classically chaotic dynamical systems.

As all realistic systems interact with their environment, the modern approach to understanding the quantum-classical transition (QCT) relies on the open system paradigm. In this paradigm, the dynamical system is not considered in isolation but analyzed taking its external interactions into account. These environmental interactions are of two types depending on whether it is possible to make measurements on the environment or not. If the environment is in principle unobservable, then the system is described by the reduced density matrix obtained from the full system-environment density matrix and tracing over the environment. If, on the other hand, certain measurements are possible on the environment, then the resulting reduced density matrix for the system depends on the results of these measurements. System evolution in this second case is therefore conditioned on the observation results. The conditioned system state, as it evolves, is said to define a quantum trajectory and inequalities governing the existence of a classical trajectory limit of quantum trajectories in continuously measured quantum systems have now been obtained. Since it yields effectively classical trajectories, we can call this pathway the strong limit of the quantum-classical transition.

If the environment is not amenable to observation, or if one decides to throw away the results of measurements on the environment—which amounts to the same thing—then the evolution of the reduced density matrix of the system is given by an unconditioned master equation. It is now no longer possible to obtain the classical trajectory limit as discussed above. One must now compare quantum and classical distributions (or, equivalently, the underlying dynamical averages) against each other: this constitutes the weak form of the quantum-classical transition. Note that for any given situation, one can always obtain the weak from the strong form of the QCT, but the reverse is not possible.

Our purpose in this work is to obtain a semiclassical analysis of the QCT for bounded, classically chaotic open systems focusing on the regularization of the singular $\hbar \to 0$ limit via the weak form of the environmental interaction, rather than the state localization characteristic of measurement. Given a small, but finite, value of $\hbar$, we aim to establish the existence of a timescale beyond which the dynamics of open quantum and classical systems becomes statistically equivalent. Note that this is quite different from the strong form of the QCT, indeed the conditions we obtain cannot be easily interpreted in that language.

The basic results for the weak form of the QCT are the following: for a bounded open system with a classically chaotic Hamiltonian, the QCT is achieved by two parallel processes. First, the semiclassical approximation for quantum dynamics, which breaks down for classically chaotic systems due to overwhelming nonlocal interference, is recovered as the environmental

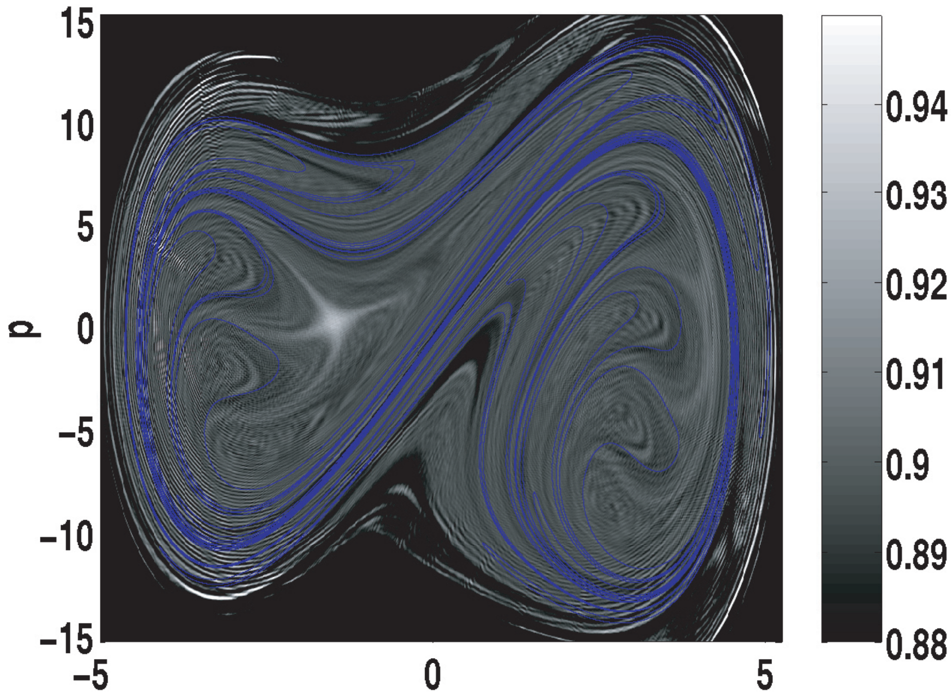

Figure 1—
The classical manifold structure superimposed on the quantum Wigner function. This structure is not visible in the evolution of the isolated system—classically or quantum mechanically—however, the diffusive effect of coupling to an environment (with or without an associated measurement process) results in the appearance of the structure in both classical and quantum evolutions. This happens when the inequalities mentioned in the text are satisfied.

interaction filters these effects. Second, the environmental noise restricts the foliation of the unstable manifold, the set of points which approach a hyperbolic point in reverse time, allowing the semiclassical wavefunction to track this modified classical geometry (see Fig. 1). Even though establishing these results requires some analysis, the final inequality is of a very simple type and can be written as a semiclassical quantization condition where the areal scale in phase space is set by the diffusion-averaged classical dynamics [1].

It is worthwhile to emphasize the topological aspects of our results. It has been known for some time that in certain systems, the evolution for classical and quantum averages is very close, while for other systems, it is not. At least for the case of chaotic systems, we have strong evidence that compactness of the phase space is a precondition for the weak form of the QCT. This issue is now under investigation.

[1] B. Greenbaum, S. Habib, K. Shizume, and B. Sundaram, quant-ph/0401174 (2004).